



The University of Sydney  
Integrated Sustainability Analysis <sup>TM</sup>



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Student topic

Estimating multi-criteria weights for  
interacting criteria from profile sets

## 1. Brief rationale

### 1.1 *Interacting criteria in multi-criteria decision analysis*

The very choice of a set of criteria against which alternatives are evaluated is a value judgment in all multi-criteria decision methods, not only with regard to which criteria are selected, but also with regard to how the criteria are defined, and in how much detail they are represented. In practical applications of multi-criteria decision methods, variables can in general not be constructed without interaction. For example, in a multi-criteria environmental impact assessment, ‘energy consumption’ and ‘greenhouse gas emissions’ have in common the CO<sub>2</sub> that results from burning fossil fuels. However, both indicators also contain mutually exclusive parts: ‘energy consumption’ comprises renewable energy sources that do not emit, or sources with varying CO<sub>2</sub> coefficients, or non-combustion fuel use such as in feedstocks or for bitumen. Similarly, ‘greenhouse gas emissions’ includes gases other than CO<sub>2</sub>, and non-energy sources such as agriculture. The results of decision analyses (for example between alternative energy sources) with interacting indicators may be contended, because issues represented in high detail by many criteria are likely to receive a disproportionately high influence.

### 1.2 *Approaches to compensate for criteria interaction*

During the past three decades, a method has been developed that provides a way of aggregating interacting criteria: The fuzzy or Choquet integral (Sugeno 1974) is essentially a distorted arithmetic mean that takes into account the importance of single criteria as well as that of sets of criteria. The solution of a fuzzy integral is quite complex, because it involves evaluating numerous combinations of sets, and the number of its weight coefficients grows exponentially with the number of criteria. For this reason, the determination of weights in practice has been an often revisited topic. At present, the main approaches are feasible only for systems where interaction exists only between a small number of criteria (so-called  $k$ -order fuzzy measures where  $k \leq 3$ ; Marichal and Roubens 2000). Applied in full generality, existing approaches suffer from prohibitive cognitive ability, data requirements or complexity.

### 1.3 *Challenge*

Develop a formulation for the weights of a fuzzy integral, using correlation matrices in order to account for criteria interaction.

### 1.4 *Application*

The report Balancing Act (<http://www.isa.org.usyd.edu.au>) lists Triple Bottom Line (TBL) accounts for 135 industry sectors of the Australian economy in terms of four financial, three social and four environmental indicators. In conjunction, the ten indicators provide a macro-landscape of the Australian economy against which many sectoral management issues can be benchmarked. The criteria interaction concept can

be applied to these 11 criteria and 135 industry sectors. The aim is to demonstrate that applying the fuzzy integral to such a multi-criteria problem can defuse debates over criteria choice and construction, and thus leads to a more acceptable outcome of industry benchmarking.

## 2. Knowledge, tasks and skills

- Combinatorics
- Linear Algebra
- Programming
- Multi-criteria decision science

## 3. Supervisor

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## 4. Literature

Sugeno 1974; Weber *et al.* 1988; Doyle 1995; Grabisch 1996a; b; Marichal 2000; Marichal and Roubens 2000; Ebert and Welsch 2004; Kojadinovic 2004

## 5. References

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